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CONSTANTS AND DIAGRAMS FOR REPEATED-STRESS CALCULATIONS.

By H. F. Moore and F. B. Seely.

SUMMARY.

This paper is supplementary to the paper presented by the writers at the annual meeting of the Society in 1915 entitled, "The Failure of Materials under Repeated Stress." A revised table of constants is given, based on further study of test data, and on the correction of a numerical error discovered in the method of deriving values of the constants given in the paper presented in 1915.

Diagrams are given for the solution of the exponential equations noted in the previous paper, and a discussion is presented of the "factor of safety" as applied to repeated-stress conditions.

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At the 1915 annual meeting of this Society the writers presented a paper entitled "The Failure of Materials under Repeated Stress."[1] In that paper the following formulas were proposed for failure of metals under repeated stress:

$$S = \frac{B}{(1-Q)N^{\frac{1}{8}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

or
$$\log S = \log B - \log (1-Q) - \tfrac{1}{8} \log N \quad \ldots\ldots\ldots (1a)$$

and
$$S = \frac{B}{(1-Q)N^{\frac{1}{8}}} (1 + 0.015\, N^{\frac{1}{8}}) \quad \ldots\ldots\ldots\ldots (2)$$

or
$$\log S = \log B - \log (1-Q) - \tfrac{1}{8} \log N + \log (1 + 0.015\, N^{\frac{1}{8}}) \quad \ldots (2a)$$

In these formulas S is the intensity of fiber stress in pounds per square inch (computed by the ordinary formulas of mechanics of materials) corresponding to failure after N repetitions of stress; Q is the ratio of minimum stress during one cycle of stress repetition to the maximum stress during the cycle; B is a constant determined experimentally for any metal. Equation (1) was recommended for use in designing members whose failure would endanger life or limb; equation (2) for cases in which danger to life or limb is not involved.

Tentative values of the constant B were determined from a study of test data, and were given in the paper above referred to. Since the presentation of that paper further study of test data has been made, and an error has been discovered in the method of computing B from test data noted therein.[2] The correct method of determining B from data of tests in which $Q = -1$ (stress completely reversed) is as follows:

[1] *Proceedings*, Am. Soc. Test. Mats., Vol. XV, Part II, p. 437 (1915).

[2] The following is quoted from the paper of last year: "B has been determined for a number of materials from the data of repeated stress tests. It is the value of the ordinate for $N = 1$ (line extended backward) and $Q = -1$ (stress completely reversed)."

On a logarithmic diagram of test results in which values of S for failure are plotted as ordinates and values of N as abscissas, the value of B is *twice* that of the ordinate for $N = 1$ (line extended backward). After correcting the error in the paper and making further study of test data, tentative values of B are submitted, as given in Table I.

For solving the complex equations for repeated stress given above, the writers present two diagrams (Figs. 1 and 2), which they have found more convenient to use than the diagrams given in the previous paper.[1] Fig. 1 gives the diagram corresponding

TABLE I.—TENTATIVE VALUES OF B.

Material.	B	Log B.
Structural Steel and Soft Machinery Steel	250 000	5.39794
Wrought Iron	250 000	5.39794
Steel, 0.45 per cent Carbon	350 000	5.54407
Cold-rolled Steel Shafting	400 000	5.60206
Tempered Spring Steel	400 000 to 800 000	5.60206 5.90309
Hard-Steel Wire	600 000	5.77815
Gray Cast Iron	100 000	5.00000
Cast Aluminum	80 000	4.90309
Hard-drawn Copper	140 000	5.14613

to equation (1) and Fig. 2, the diagram corresponding to equation (2). The method of using either diagram is as follows:

Enter the diagram at the lower edge with the desired value of N as abscissa; pass vertically to the diagonal line for the value of B for the given material (if the exact value of B for the material is not plotted the location of its line can be judged by interpolation with a good degree of accuracy); then pass horizontally to the diagonal line for the value of Q corresponding to the given range of stress; then vertically to the upper edge of the diagram where the value of S for fatigue failure may be read from the scale. It should always be kept in mind that in

[1] Attention is called to the alignment charts for the solution of equation (1) given by Prof. J. B. Kommers, of the University of Wisconsin, in the *American Machinist* for Feb. 3, 1916.

no case is S to be taken greater than the static strength of the material.

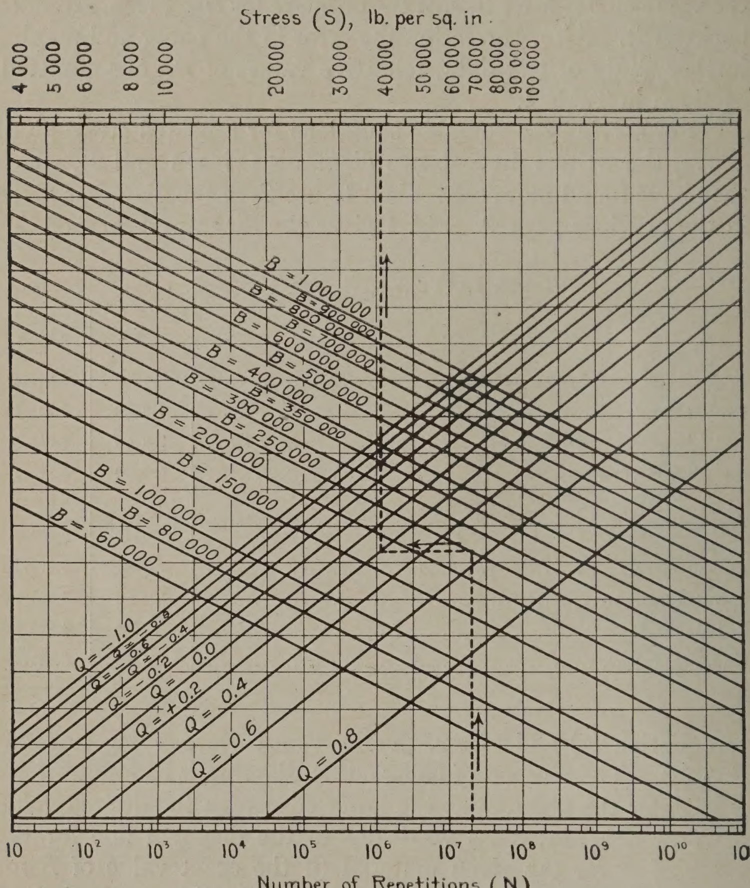

FIG. 1.—Diagram for Solution of Equation (1), $S=\dfrac{B}{(1-Q)N^{\frac{1}{8}}}$.

Example:

In Fig. 1 the dotted lines indicate the solution of the following problem: Value of B for the material (structural steel), 250,000; $Q=+0.2$; $N=20{,}000{,}000$; find S. *Ans.*—$S=38{,}000$ lb. per sq. in. and static strength would govern.

In Fig. 2 the dotted lines indicate the solution of the following problem: Value of B, 400,000; $Q=-1.0$; $N=12{,}000{,}000{,}000$; find S. *Ans.*—$S=$ 14,400 lb. sq. in.

In studying repeated-stress test data, or in choosing working stresses for members subjected to repeated stress, it should be remembered that a small reduction in stress very greatly

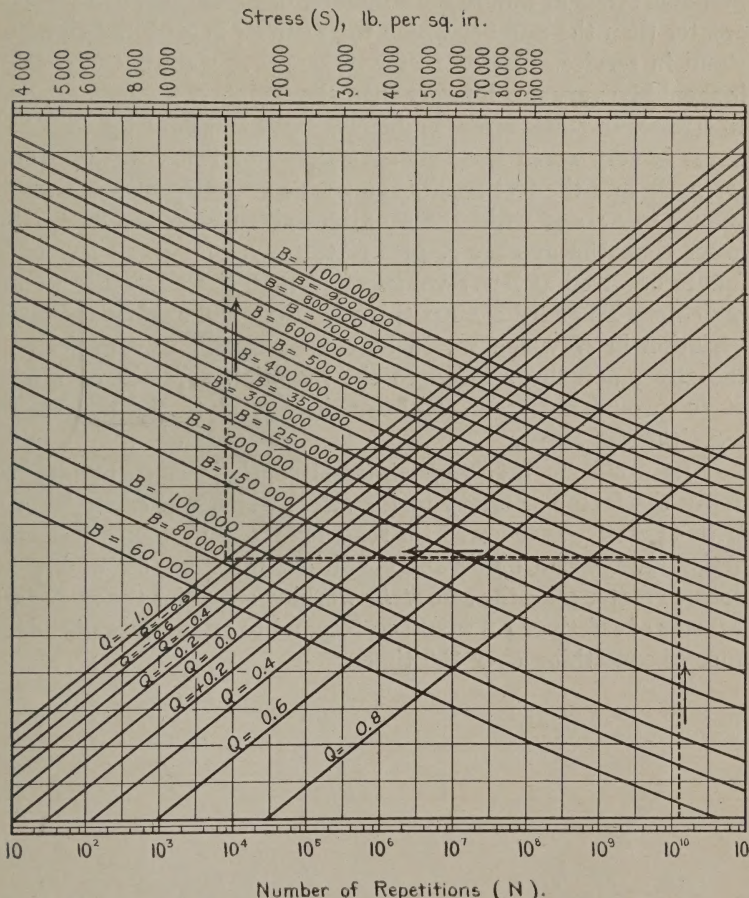

FIG. 2.—Diagram for Solution of Equation (2), $S=\frac{B}{(1-Q)N^{\frac{1}{8}}}(1+0.015N^{\frac{1}{3}})$.

increases the number of repetitions of stress which the member can carry. For metals a decrease of 9 per cent in stress about doubles the endurance, so far as can be determined from test data. Since the number of repetitions is so sensitive to changes

in the magnitude of stress, it seems more logical for repeated-stress problems to apply the "factor of safety" used to the number of repetitions rather than to the stress, computing the probable stress at failure for a number of repetitions many times greater than the number which the member is expected to withstand in service. If the factor of safety is applied to N, then it should be very much larger than the factors commonly applied to stresses in static-stress problems. Our test data for repeated stress is very much less extensive than our test data for static strength, and the test results show wide variation in N for small change in S; and while the equations given yield results a little lower than the average results of tests, yet for some few tests failure occurred at lower values of N than the equations would indicate. To guard against this variation, and against the large variation in N which would be caused by slight variations in the stress actually applied to the material, a factor of safety of 100 would not seem too large, if applied to the number of repetitions of stress.

In problems involving static strength it is customary to apply the factor of safety to the stress which will cause failure. If this practice is followed for repeated-stress problems (remembering always that under any conditions the stress allowed must not be greater than the safe *static* working stress for the material) a factor of safety of 1.8 would correspond to an increase of endurance of something over 100 times.